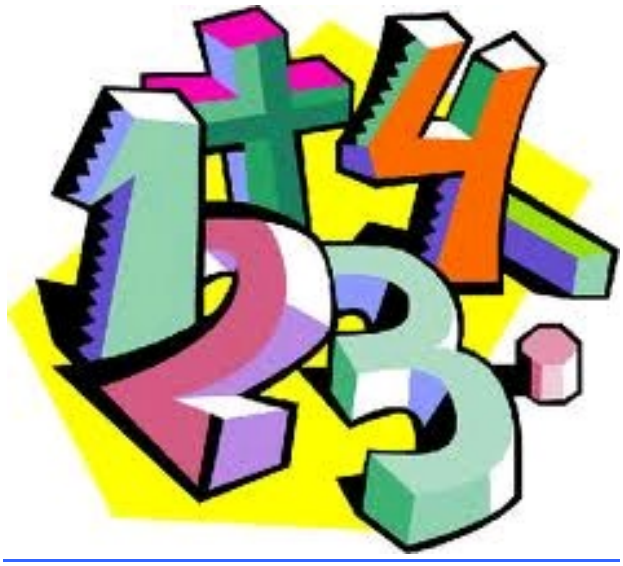
VCE INDUCTION PACKAGE 2021/2022

UNITS 1 AND 2

MATHEMATICAL METHODS



Welcome letter

Dear Students,

Welcome to Mathematical Methods Units 1 and 2.

If you are well organised, motivated and have a good work ethic, you will have an enjoyable and successful year in Mathematical Methods .

In order to get the most out of this course, there are a number of things that we strongly suggest that you do over the summer and continue throughout 2022.

1. Complete the transition exercises by the start of the 2022 academic year.

2. Familiarise yourself with the detailed course outline provided in this package.

3. Develop a good working relationship with your class teacher and maintain regular communication with them throughout the year.

4. Develop a study timetable that will assist you in meeting the work deadlines.

5. Ensure that you become familiar with the resources (prescribed textbook, websites, notes, other textbooks), and always have the required material with you in class.

6. Become familiar with the school's VCE compliance policy by reading the VCE handbook located on MOODLE.

7. Communicate with students who have studied the subject in previous years to get their perspective and suggestions for success.

8. Ensure that you have a balanced life that consists of schoolwork, exercise, sport, leisure, rest and a healthy diet.

If you have any queries about the course, please contact me at school or by email.

On behalf of the Mathematical Methods Study staff, we wish you all the best for your studies next year,
Regards,

Mr Keatch, Ms Neilson, Mr Shannon and Mr Spiezia

Email addresses/Staffroom locations of Staff teaching subject

NAME OF TEACHER	EMAIL ADDRESS	LOCATION
Mr Keatch	keatcha@vermontsc.vic.edu.au	Main staff room
Ms Neilson	neilsonr@vermontsc.vic.edu.au	Daily Organiser office
Mr Shannon	shannonj@vermontsc.vic.edu.au	Senior SM/HoH office
Mr Spiezia	spiezial@vermontsc.vic.edu.au	Main staff room

Outcomes for Units 1 & 2

Outcome 1

On completion of each unit the student should be able to define and explain key concepts as specified in the content from the areas of study, and apply a range of related mathematical routines and procedures.
To achieve this outcome the student will draw on knowledge and skills outlined in all the areas of study.

Outcome 2

On completion of each unit the student should be able to apply mathematical processes in non-routine contexts, including situations requiring problem-solving, modelling or investigative techniques or approaches, and analyse and discuss these applications of mathematics.
To achieve this outcome the student will draw on knowledge and skills outlined in one or more areas of study.

Outcome 3

On completion of each unit the student should be able to use numerical, graphical, symbolic and statistical functionalities of technology to develop mathematical ideas, produce results and carry out analysis in situations requiring problem-solving, modelling or investigative techniques or approaches.
To achieve this outcome the student will draw on knowledge and skills outlined in all the areas of study.

Note : The use of technology should be developed as an integral part of the range of learning activities for Outcomes 1 & 2

Areas of Study – Unit 1

Area of Study 1
Functions and graphs

In this area of study students cover the graphical representation of simple algebraic functions (polynomial and power functions) of a single real variable and the key features of functions and their graphs such as axis intercepts, domain (including the concept of maximal, natural or implied domain), co-domain and range, stationary points, asymptotic behaviour and symmetry. The behaviour of functions and their graphs is explored in a variety of modelling contexts and theoretical investigations.
This area of study includes:

- review of coordinate geometry
- functions and function notation, domain, co-domain and range, representation of a function by rule, graph and table
- use of the vertical line test to determine whether a relation is a function or not, including examples of relations that are not functions and their graphs such as $x = k$, $x = ay^2$ and circles in the form $(x - h)^2 + (y - k)^2 = r^2$
- qualitative interpretation of features of graphs of functions, including those of real data not explicitly represented by a rule, with approximate location of stationary points
- graphs of power functions $f(x) = x^n$ for $n \in N$ and $n \in \{-2, -1, 1{,}3\ ,\ 1,\ 2\}$, and transformations of these graphs to the form $y = a(x + b)^n + c$ where $a, b, c \in R$ and $a \neq 0$
- graphs of polynomial functions to degree 4 and other polynomials of higher degree such as $g(x) = (x + 2)^2(x - 1)^3 + 10$
- graphs of inverse functions.

Area of Study 2
Algebra

This area of study supports students' work in the 'Functions and graphs', 'Calculus' and 'Probability and statistics' areas of study, and content is to be distributed between Units 1 and 2. In Unit 1 the focus is on the algebra of polynomial functions of low degree and transformations of the plane.

This area of study includes:

• use of symbolic notation to develop algebraic expressions and represent functions, relations, equations and systems of simultaneous equations

• substitution into and manipulation of these expressions

• recognition of equivalent expressions and simplification of algebraic expressions involving different forms of polynomial and power functions, the use of distributive and exponent laws applied to these functions, and manipulation from one form of expression to an equivalent form, including expansion of $(x + a)^n$ where $n \in N$

• use of parameters to represent families of functions and determination of rules of simple functions and relations from given information

• transformations of the plane and application to basic functions and relations by simple combinations of dilations (students should be familiar with both 'parallel to an axis' and 'from an axis' descriptions), reflections in an axis and translations, including the use of matrices for transformations

• the connection between the roots of a polynomial function, its factors and the horizontal axis intercepts of its graph, including the remainder, factor and rational root theorems

• solution of polynomial equations of low degree, numerically (including numerical approximation of roots of simple polynomial functions using bisection), graphically and algebraically

• solution of a set of simultaneous linear equations (geometric interpretation only required for two variables) and equations of the form $f(x) = g(x)$ numerically, graphically and algebraically.

Area of Study 3
Calculus

In this area of study students cover constant and average rates of change and an introduction to instantaneous rate of change of a function in familiar contexts, including graphical and numerical approaches to estimating and approximating these rates of change.

This area of study includes:

• average and instantaneous rates of change in a variety of practical contexts and informal treatment of instantaneous rate of change as a limiting case of the average rate of change

• interpretation of graphs of empirical data with respect to rate of change such as temperature or pollution levels over time, motion graphs and the height of water in containers of different shapes that are being filled at a constant rate, with informal consideration of continuity and smoothness

• use of gradient of a tangent at a point on the graph of a function to describe and measure instantaneous rate of change of the function, including consideration of where the rate of change is positive, negative, or zero, and the relationship of the gradient function to features of the graph of the original function.

Area of Study 4
Probability and statistics

In this area of study students cover the concepts of event, frequency, probability and representation of finite sample spaces and events using various forms such as lists, grids, venn diagrams, karnaugh maps, tables and tree diagrams. This includes consideration of impossible, certain, complementary, mutually exclusive, conditional and independent events involving one, two or three events (as applicable), including rules for computation of probabilities for compound events.

This area of study includes:

• random experiments, sample spaces, outcomes, elementary and compound events

• simulation using simple random generators such as coins, dice, spinners and pseudo-random generators using technology, and the display and interpretation of results, including informal consideration of proportions in samples

• probability of elementary and compound events and their representation as lists, grids, venn diagrams, karnaugh maps, tables and tree diagrams

• the addition rule for probabilities, $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$, and the relation that for mutually exclusive events $\Pr(A \cap B) = 0$, hence $\Pr(A \cup B) = \Pr(A) + \Pr(B)$

• conditional probability in terms of reduced sample space, the relations $\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$ and

$\Pr(A \cap B) = \Pr(A \mid B) \times \Pr(B)$

• the law of total probability for two events $\Pr(A) = \Pr(A \mid B) \Pr(B) + \Pr(A \mid B')\Pr(B')$

• the relations that for pairwise independent events A and B, $\Pr(A \mid B) = \Pr(A)$, $\Pr(B \mid A) = \Pr(B)$ and $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$.

Areas of Study – Unit 2

Area of Study 1
Functions and graphs

In this area of study students cover graphical representation of functions of a single real variable and the key features of graphs of functions such as axis intercepts, domain (including maximal, natural or implied domain), co-domain and range, asymptotic behaviour, periodicity and symmetry.
This area of study includes:

• review of trigonometry (sine and cosine rules not required)

• the unit circle, radians, arc length and conversion between radian and degree measures of angle

• sine, cosine and tangent as functions of a real variable, and the relationships $\sin(x) \approx x$ for small values of x, $\sin^2(x) + \cos^2(x) = 1$ and $\tan(x) = \frac{\sin(x)}{\cos(x)}$

• exact values for sine, cosine and tangent of $\frac{n\pi}{6}$ and $\frac{n\pi}{4}$, $n \in Z$

• symmetry properties, complementary relations and periodicity properties for sine, cosine and tangent functions

• circular functions of the form $y = af(bx) + c$ and their graphs, where f is the sine, cosine or tangent function, and $a, b, c \in R$ with $a, b \neq 0$

• simple applications of sine and cosine functions of the above form, with examples from various modelling contexts, the interpretation of period, amplitude and mean value in these contexts and their relationship to the parameters a, b and c

• exponential functions of the form $f: R \to R, f(x) = Aa^{kx} + C$ and their graphs, where $a \in R^+$, $A, k, C \in R, A \neq 0$

• logarithmic functions of the form $f: R_+ \to R, f(x) = \log_a(x)$, where $a > 1$, and their graphs, as the inverse function of $y = a^x$, including the relationships $a^{\log_a(x)} = x$ and $\log_a(a^x) = x$

• simple applications of exponential functions of the above form, with examples from various modelling contexts, and the interpretation of initial value, rate of growth or decay, half-life and long run value in these contexts and their relationship to the parameters A, k and C.

Area of Study 2
Algebra

This area of study supports students' work in the 'Functions and graphs', 'Calculus' and 'Probability and statistics' areas of study. In Unit 2 the focus is on the algebra of some simple transcendental functions and transformations of the plane. This area of study provides an opportunity for the revision, further development and application of content prescribed in Unit 1, as well as the study of additional algebra material introduced in the other areas of study in Unit 2 as follows:

• use of inverse functions and transformations to solve equations of the form $Af(bx) + c = k$, where $A, b, c, k \in R$ and $A, b \neq 0$ and f is sine, cosine, tangent or a^x, using exact or approximate values on a given domain

• index (exponent) laws and logarithm laws, including their application to the solution of simple exponential equations

• numerical approximation of roots of cubic polynomial functions using Newton's method.

Area of Study 3
Calculus

In this area of study students cover first principles approach to differentiation, differentiation and anti-differentiation of polynomial functions and power functions by rule, and related applications including the analysis of graphs.

This area of study includes:

• graphical and numerical approaches to approximating the value of the gradient function for simple polynomial functions and power functions at points in the domain of the function

• the derivative as the gradient of the graph of a function at a point and its representation by a gradient function

• notations for the derivative of a function: $f'(x)$, $\frac{dy}{dx}$, $\frac{d}{dx}(f(x))$, $D_x(f)$

• first principles approach to differentiation of $f(x) = x^n$, $n \in Z$, and simple polynomial functions

• derivatives of simple power functions and polynomial functions by rule

• applications of differentiation, including finding instantaneous rates of change, stationary values of functions, local maxima or minima, points of inflection, analysing graphs of functions, solving maximum and minimum problems and solving simple problems involving straight-line motion

• notations for an anti-derivative, primitive or indefinite integral of a function: $F(x)$, $\int f(x)dx$

• use of a boundary condition to determine a specific anti-derivative of a given function

• anti-differentiation as the inverse process of differentiation and identification of families of curves with the same gradient function, including application of anti-differentiation to solving simple problems involving straight-line motion.

Area of Study 4
Probability and statistics

In this area of study students cover introductory counting principles and techniques and their application to probability and the law of total probability in the case of two events.

This area of study includes:

• addition and multiplication principles for counting

• combinations: concept of a selection and computation of ${}^nC_r$ application of counting techniques to probability.

The topic outline for Unit 1 can be found in the **Curriculum Grid 2022** document which can be found on MOODLE.

Assessment

The award of satisfactory completion for a unit is based on whether the student has demonstrated the set of outcomes specified for the unit. Teachers should use a variety of learning activities and assessment tasks that provide a range of opportunities for students to demonstrate the key knowledge and key skills in the outcomes. The areas of study, including the key knowledge and key skills listed for the outcomes, should be used for course design and the development of learning activities and assessment tasks. Assessment must be a part of the regular teaching and learning program and should be completed mainly in class and within a limited timeframe.

All assessments at Units 1 and 2 are school-based. Procedures for assessment of levels of achievement in Units 1 and 2 are a matter for school decision. Assessment tasks must include components to be completed with and without the use of technology as applicable to the outcomes.

Demonstration of achievement of Outcome 1 should be based on the student's performance on a selection of the following assessment tasks:

• assignments

• tests

• summary or review notes.

Demonstration of achievement of Outcome 2 should be based on the student's performance on a selection of the following assessment tasks:

• modelling tasks

• problem-solving tasks

• mathematical investigations.

Demonstration of achievement of Outcome 3 should be based on the student's performance on aspects of tasks completed in demonstrating achievement of Outcomes 1 and 2 that incorporate opportunity for the effective and appropriate use of technology.

The timing of Assessment tasks is outlined in the **Scope and Sequence 2022** document, which can be found on MOODLE

Coursework Requirements

For an outcome to be scored, complete the prescribed coursework questions **before** the date of the outcome.

Satisfactory Requirements

To obtain an S for each unit students are to;

- Attend a minimum of 90% of classes
- Complete the coursework
- Obtain at least 40% on all outcomes.

GRADING OF TASKS

Grade	UG	E	E^+	D	D^+	C	C^+	B	B^+	A	A^+
%	0-39	40-44	45-49	50-54	55-59	60-69	70-74	75-79	80-84	85-89	90-100

← **NOT adequately prepared** for Year 12 Mathematical Methods → ← Adequately prepared for Year 12 Mathematical Methods →

- There will be no retesting of assessment tasks.
- Students who receive a UG on any assessment tasks MUST demonstrate satisfactory achievement of all Outcomes in the prescribed redemption task and/or the Semester Examinations to gain an 'S' for the unit.
- Students who are absent on the day/s of an assessment task are required to organize completion of the task with their Student Manager. A medical certificate will be required if students stay away from school due to illness.

USE OF CLASS TIME

- Student behaviour which disrupts the learning of others cannot be tolerated at any time, and hence students will be exited to work in the study hall.
- Students not making effective use of their class time will be exited to work in the study hall.

Sources of support for the Study – inc. key staff, websites, documentation

Links

VCAA Mathematical Methods
http://www.vcaa.vic.edu.au/vce/studies/mathematics/methods/methmathindex.html

Useful Websites

Cambridge Senior Mathematics	https://seniormaths.cambridge.edu.au/
VCAA	www.vcaa.vic.edu.au
The Mathematical Association of Victoria	www.mav.vic.edu.au
Classpad Help Series	http://www.classpad.com.au/

Materials Required – Text, Stationery, and other Resources

Required Materials to be brought to each class

- Class notes folder / worksheet booklet and writing materials
- The text, 'Cambridge MATHEMATICAL METHODS Units 1&2 '
- Casio ClassPad CAS Calculator
- Exercise book for class work
- Display Folder for handouts / topic tests
- Non-spiral bound reference book for notes which may be used in all assessment tasks

Transition Tasks to be completed in preparation for the beginning of the 2022 school year

Task 1: Complete assigned Chapter 1 text book exercises, and the following "Questions based on Assumed Knowledge" in your workbook

Task 2: Complete the "Linear Modelling" holiday homework booklet, to be handed in at the start of Term 1 2021

Task 3: Log on to the Cambridge Senior Mathematics website and become familiar with the resources it provides

Task 4: Ensure your CAS Calculator is fully operational.

Task 5: Visit the Classpad Help Series website http://www.classpad.com.au/ which is useful for both the 'new' and 'old' calculators.

Questions based on Assumed Knowledge

Show working for the non-CAS questions!

1: Expand the following expressions

(a)	$(x + 2)(x + 3)$	(f)	$(2 + 3t)^2$
(b)	$(2x + 5)(7x – 6)$	(g)	$(7x – 2y)(3x – 4y)$
(c)	$(5e – 1)^2$	(h)	$(3 + 8x)(3 – 8x)$
(d)	$(p – 3q)(p – 7q)$	(i)	$(1 – 3x)(5x – 4)$
(e)	$(4y – 9)(4y + 9)$	(j)	$5(2t + 7)(t – 11)$

2: Factorise the following expressions

(a)	$3x^2 - 9x$	(f)	$3x^2 + 6x - 45$	(k)	$-x^2 + 6x - 8$
(b)	$x^2 + 2x - 15$	(g)	$3x^2 + 8x + 4$	(l)	$27 - 12x^2$
(c)	$y^2 + 5y + 6$	(h)	$7y^2 - 10y + 3$	(m)	$2x^2 - 28x + 98$
(d)	$m^2 - 4m - 21$	(i)	$25e^2 - 16f^2$	(n)	$6x^2 - 11x - 72$
(e)	$x^2 - 64$	(j)	$3y^2 - 4y - 20$	(o)	$x^2 + 5xy - 24y^2$

3: Factorise the following expressions

(a) $x^2 + 4x + 1$

(b) $x^2 - 6x - 4$

(c) $x^2 + 12x + 15$

(d) $x^2 - 10x - 11$

(e) $x^2 - 8x - 2$

(f) $x^2 + 6x + 11$

CAS PRACTICE EXERCISES

Use the CAS calculator to do the following questions. Answers to 2 decimal places where necessary.

26) Find the y-intercept of the graph with equation $6x - 5y + 18 = 0$

27) Solve the following equations:

(a) $5 + \frac{2x}{7} = 7$ (b) $4 - 3(4a + 5) = 5(7 - 4a)$ (c) $\frac{2k - 5}{3} - 2 = \frac{4 - k}{2}$

28) Find the x-intercepts of the parabola with equation $y = \frac{4x^2}{5} - 2x - 3$

29) Find the turning point of the parabola **in** the previous question.

30) Transpose the following equation to make a the subject:

$$\frac{3ax + 7}{2} + 8 = \frac{5ax - 4}{2} - b$$

31) Find the gradient and y-intercept of the line with equation $\frac{5x}{8} - 6y = 15$

32) Find the length of the line joining the points (-6,27) and (14,-12)

33) Find the coordinates of the points on the curve with equation $y = x^3 - 2x^2 + 5$ at which the y-value is -3.

34) (a) Find the equation of the line joining the points (-6,8) and (2,2)
(b) Now find the y-value of the point on this line where x = -3

Answers

1:

(a) $x^2 + 5x + 6$ (f) $4 + 12t + 9t^2$ or $9t^2 + 12t + 4$

(b) $14x^2 + 23x - 30$ (g) $21x^2 - 34xy + 8y^2$

(c) $25e^2 - 10e + 1$ (h) $9 - 64x^2$

(d) $p^2 - 10pq + 21q^2$ (i) $-15x^2 + 17x - 4$

(e) $16y^2 - 81$ (j) $10t^2 - 75t - 385$

2: (a) $3x(x - 3)$ (f) $3(x + 5)(x - 3)$ (k) $-(x - 4)(x - 2)$

(b) $(x + 5)(x - 3)$ (g) $(3x + 2)(x + 2)$ (l) $3(3 - 2x)(3 + 2x)$

(c) $(y + 3)(y + 2)$ (h) $(7y - 3)(y - 1)$ (m) $2(x - 7)^2$

(d) $(m - 7)(m + 3)$ (i) $(5e - 4f)(5e + 4f)$ (n) $(3x + 8)(2x - 9)$

(e) $(x - 8)(x + 8)$ (j) $(3y - 10)(y + 2)$ (o) $(x + 8y)(x - 3y)$

3: (a) $(x + 2 - \sqrt{3})(x + 2 + \sqrt{3})$

(b) $(x - 3 - \sqrt{13})(x - 3 + \sqrt{13})$

(c) $(x + 6 - \sqrt{21})(x + 6 + \sqrt{21})$

(d) $(x - 11)(x + 1)$

did you try the quick way first?

(e) $(x - 4 - \sqrt{18})(x - 4 + \sqrt{18}) = (x - 4 - 3\sqrt{2})(x - 4 + 3\sqrt{2})$

the surd should be simplified if possible!

(f) cannot be factorised ...why?

26: (0 , 3.6)

27 a) $x = 7$

b) $a = 5\frac{3}{4}$

c) $k = 4\frac{6}{7}$

28: (-1.05 , 0) and (3.55 , 0)

29: (1.25 , -4.25)

30: $a = \frac{2b + 7}{2x}$

31: y-int = (0 , -2.5) $m = \frac{5}{48}$

32: 43.83 units

33: (-1.24 , -3) & ((2 , -3) & (3.24 , -3)

34: a) $y = -0.75x + 3.5$

b) $y = 5.75$